Exercise 12

For the following exercises, find the average rate of change of each function on the interval specified for real numbers b or h.

$$b(x) = \frac{1}{x+3} \text{ on } [1, 1+h]$$

Solution

The average rate of change of the function on $[1, 1+h]$ is

$$\begin{aligned}
\frac{b(1+h)-b(1)}{(1+h)-1} &= \frac{\frac{1}{(1+h)+3} - \frac{1}{(1)+3}}{h} \\
&= \frac{\frac{1}{h+4} - \frac{1}{4}}{h} \\
&= \frac{\frac{4}{4(h+4)} - \frac{h+4}{4(h+4)}}{h} \\
&= \frac{\frac{4-(h+4)}{4(h+4)}}{h} \\
&= \frac{4-h-4}{4h(h+4)} \\
&= \frac{-h}{4h(h+4)} \\
&= -\frac{1}{4(h+4)}.
\end{aligned}$$